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A RANDOM ACCESS ALGORITHM FOR
FREQUENCY HOPPED SPREAD SPECTRUM PACKET
RADIO NETWORKS

Michael Georgiopoulos and P. Papantoni-Kazakos

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Abstract

We propose and analyze a limited sensing random access algorithm, named CRAFH, for a multi-user and multi-receiver frequency hopped spread spectrum system. Utilizing the regenerative character of the induced by the algorithm transmission process, we compute throughputs and expected per packet delays. In the presence of interferences between transmissions to different receivers, we compute throughputs, subject to an upper bound on the probability of erroneous data decoding. The CRAFH induces uniformly good delays within its stability region, and is particularly appropriate for environments where the users are highly mobile.

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1. Introduction

The authors ~~We~~ consider packet radio multi-user spread-spectrum environments, where frequency hopping spread spectrum techniques are deployed for protection against intelligent adversaries[1]. When the users in such environments are mobile and bursty, random access frequency hopping transmission algorithms should be adopted, for efficiency in throughput and delay control. In this paper, we proposes and analyzes such an algorithm, named Collision Resolution Algorithm for Frequency Hopping (CRAFH). The CRAFH is a limited sensing random access algorithm utilizing receiver oriented frequency hopping patterns. In its design, the experience from random access algorithms for non spread spectrum multi-user channels is utilized. See p. a

For the Poisson user model (large number of independent bursty users), and for transmissions through a single non spread spectrum channel with feedback, the existing stable random access algorithms belong to two distinct classes: The full sensing class, and the limited sensing class. The former requires that each user know the overall feedback history, and it includes algorithms such as those in [2], [3], and [4]. The latter requires that each user tune to the feedback broadcast only while he is blocked (from the time he generates a message to the time this message is successfully transmitted), and it includes the algorithms in[5], [6], [7], [15], [16], and [17]. In mobile user environments, only the limited sensing class of random access algorithms is applicable, since the users can not then tune to the feedback broadcast whenever they move off the broadcast range.

Regarding packet radio multi-user frequency hopping, Hajek [10] studied a full sensing random access algorithm, for the Poisson user model and a single frequency hopping channel. Pursley and Geraniotis ([8] and [9]) studied the probability of error induced, when no random access algorithm is deployed.

2. System Model

We consider the case where a large number of mobile independent bursty users use distinct frequency hopping patterns, to transmit to a given number, N_R, of

semistatic receivers, through a common channel. We then model the overall user traffic as Poisson, and we also assume packet users and fixed length packets. In addition, we consider a synchronous system, where the time of the common channel is divided into disjoint consecutive slots, each of length equal to one packet, and where a packet transmission can only start at the beginning of some slot. We then measure time in slot units, where slot T occupies the time interval [T,T+1). We assume that each newly generated packet is destined for receiver k, $1 \leq k \leq N_R$, with probability $1/N_R$. Thus, if λ_o is the intensity of the overall Poisson user traffic, then the traffic per receiver is also Poisson, with intensity $\lambda=\lambda_o/N_R$. Finally, we assume that the maximum delay with which some transmission from any user reaches any of the N_R receivers is α, where $\alpha << 1$.

The bandwidth of the common transmission channel is divided into q orthogonal frequency bins, where each bin is uniquely identified by its central frequency. At the beginning of each slot, a distinct frequency hopping pattern is assigned to each one of the N_R receivers, where each such pattern consists of some of the above frequency bins. We will assume that q equals a power of some prime number, and that $q \geq N_R$, and we will then adopt the frequency hopping patterns derived from the Reed-Solomon code [11]. Then, (i) The length, m, of each frequency hopping pattern is such that $m \leq q-1$. (ii) Each frequency hopping pattern contains a frequency bin at most once. (iii) Any two frequency hopping patterns have at most one common frequency bin, all cyclic shifts considered. Let $\{f_i^{(k)}\ ;\ 1 \leq i \leq m\}$, $1 \leq k \leq N_R$, denote the frequency hopping pattern assigned to receiver k at the beginning of slot T. This assignment is known to all users and all receivers. If some user wishes to send a packet to receiver k in slot T, he divides the packet into M equal length bytes, each containing at least one bit, and transmits the ith byte at the frequency bin $f_{i\,mod\,m}^{(k)}$. The above is a slow frequency hopped system, and the duration, A, of a byte is an integer multiple of the bit duration. Also, we assume that the BFSK (binary frequency shift keying) modulation scheme is deployed; thus, each frequency

bin consists of two orthogonal tone positions. The byte duration, A, corresponds to a hop interval.

We will initially adopt the following assumption, which leads to N_R single-receiver multi-user decoupled systems, and which will be relaxed later in the paper.

Assumption A1. Simultaneous transmissions to different receivers do not interfer.

If assumption A1 is true, then the overall system consists of N_R indentical and independent single receiver systems, each with input traffic Poisson intensity λ. For each such system, we assume noiseless transmission channel*, and feedback broadcast capabilities. In particular, we distinguish between the following events:

(i) The absence of a packet within some slot is detected by the receiver from the absence of signal over the corresponding frequency hopping pattern. This outcome, E, is then broadcasted to the users.

(ii) When a single packet is transmitted within a slot, the receiver reads it correctly by locking on to it. The receiver broadcasts then this success event, S.

(iii) If at least two packets are simultaneously transmitted within a slot, then depending on the corresponding delays with which these packets reach the receiver, either one packet is captured, or the information included in all the packets is destroyed. In the first case, the outcome S (success) is broadcasted by the receiver. In the second case, the event C(collision)is broadcasted instead.

The events E,C, and S are broadcasted in distinct per event and per receiver codes. Each user in the overall system, as well as the remaining receivers receive those broadcasts with maximum delay α. The event S includes information on some differential delays. This information will be explicitly stated as part of the CRAFH algorithm, in section 3. Let n be the nonnegative integer such that $nA \leq \alpha < (n+1)A$, where A is the hop interval. We will then assume that $n < m$, where m is the number of frequencies in the hopping patterns. Let then $K \geq 2$ packets be simultaneously transmitted within the same slot, and let them reach the receiver with delays, $d_1, \ldots, d_K$, where $d_1 < d_2 \leq \ldots \leq d_K \leq \alpha$. Let in addition $d_j - d_1 \geq A$, for all $j \neq 1$, where A is the hop interval. From the Reed-Solomon frequency hopping patterns, it can be then seen that the first arrived packet occupies frequency bins that are orthogonal to those occupied

* Algorithms designed for noiseless transmission channels can be studied in the presence of noise, as in [6] and [7].

by the remaining packets; thus the former packet is then captured by the receiver. Thus, in the case of multiple transmissions within the same slot, we define:

> Capture. A packet is captured, if it reaches the receiver first, and its differential delay from the second arriving packet is at least equal to the hop interval A.

Otherwise, the multiple transmissions result in collision.

We note that if $\alpha < A$, the event of capture never occurs. From now on, we will assume that $\alpha = nA$, where n is an integer such that, $2 \leq n \leq m$. We will also adopt the realistic, even in the presence of high mobility, assumption that the spatial placement of the users remains unchanged within a single slot period. Finally, we will assume that all users and receivers in the system possess clocks, which remain synchronized with accuracy A (hop interval). This can be accomplished if, for example, synchronization information is included in the broadcasts of the receivers.

3. The Description of the CRAFH Algorithm

Due to the existence of clocks and the unchanged spatial placement of the users within a slot period, a user who transmits in some slot T, can compute the delay, d, between the time instant T, and the time when he receives from the addressed receiver the feedback corresponding to slot T. At the same time, the receiver can also compute d, as the difference between the time when slot T begins, and the arrival time of the packet. In the case of a single transmission within the slot T, the receiver computes the delay d of the packet, where $\ell A \leq d < (\ell+1)A$; $\ell \geq 0$, and broadcasts the integer ℓ in the place of S (success). In the case of multiple transmissions within the slot T, with corresponding delays $d_1 < d_2 \leq \ldots$, such that $d_j - d_1 \geq A$; $\forall j \geq 2$ and $\ell_1 A \leq d_1 < (\ell_1+1)A$; $\ell_1 \geq 0$, the receiver captures the first arrived packet, and broadcasts the integer ℓ_1 in the place of S (success). The above integers ℓ and ℓ_1 are encoded and then broadcasted, via frequency hopping patterns that are orthogonal to those used for packet transmissions. Upon receiving the latter broadcasts, the corresponding users can identify their success by comparing the numbers ℓ or ℓ_1 to their own precomputed delays d (in particular to the number n, such that $nA \leq d < (n+1)A$),

and the corresponding successfully transmitted packets depart then the system. We note that in the case of a capture event, there is only one packet that corresponds to the broadcasted integer ℓ_1. Upon receiving the latter broadcast, the remaining packets can then identify their failure, by comparing ℓ_1 to their own precomputed delays d.

Subject to assumption A1, let us consider the single receiver system. Let x_T denote the broadcast that corresponds to slot T, where x_T equals either E, or C, or S. The CRAFH is then implemented by each user independently, as follows:

I. Each user initiates the algorithm at the time instant when he generates a new packet. He follows the rules of the algorithm until this packet is successfully transmitted, observing simultaneously the feedbacks broadcasted by the receiver. In the implementation of the algorithm the user uses a counter, whose value r_T at slot T is a nonnegative integer. The user transmits the packet in slot T, if and only if $r_T=0$. The values of the counter are updated, as follows:

I.1. If the new packet is generated in $[T-1,T)$, then $r_T=0$.

I.2. If $r_T=0$ and $x_T=S$, then,

I.2.a. If the user identifies success for himself, then the packet is successfully transmitted and the algorithm stops.

I.2.b. If the user identifies failure for himself, then he sets, $r_{T+1}=0$.

I.3. If $r_T=0$ and $x_T=C$, then,

$$r_{T+1} = \begin{cases} 0 \ ; \ \text{w.p. } 0.5 \\ 1 \ ; \ \text{w.p. } 0.5 \end{cases}$$

I.4. If $r_T \geq 1$ and $x_T=S$, then $r_{T+1}=r_T$.

I.5. If $r_T \geq 1$ and $x_T=E$, then $r_{T+1}=r_T-1$.

I.6. If $r_T \geq 1$ and $x_T=C$, then $r_{T+1}=r_T+1$.

4. Analysis of the Algorithm

In this section, we will study the performance of the CRAFH, subject to

assumption A1 in section 2. The analysis is facilitated by the concept of a marker. The marker can be seen as an outside observer, who uses a counter. Denoting the value of this counter at slot T, R_T, then at time zero when the system operation begins, we set $R_0=1$. After that, the values of R_T are updated as determined by the rules I.4, I.5, and I.6 of the algorithm, in section 3, until the first time, T', that $R_{T'}=0$. The time T' determines the end of the first session, as induced by the algorithm. Then, at T', $R_{T'}$ is set equal to one, the second session begins, and the above process is repeated. It can be easily seen that the session lengths are i.i.d. random variables. Considering the single receiver system, let λ be the intensity of the Poisson traffic addressing the receiver, and let us define,

p_k : The probability of capture, given that $k, k\geq 2$, packets are transmitted within a single slot.

L_k : The expected length of a session, given that it starts with k packet transmissions.

L : The expected length of a session.

Assuming, as in section 2, that $\alpha=nA$; $n\geq 2$, and considering highly mobile users, where in any slot time interval the users are randomly and uniformly spatially distributed, we conclude that the delays to the receiver in slot T are uniformly distributed in [T,T+nA), and that they are independent for different slots. Then,

$$p_k = (1-n^{-1})^k \quad ; \quad k\geq 2,\ n\geq 2 \tag{1}$$

Also, we trivially conclude that,

$$L = \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} L_k \tag{2}$$

Let us define, for p_k as in (1), and $U(x)=\begin{cases}1 \; ; \; x\geq 0\\ 0 \; ; \; x<0\end{cases}$,

$$d_{1\ell} \triangleq e^{-\lambda} \frac{\lambda^\ell}{\ell!} \quad ; \quad \ell\geq 0 \tag{3}$$

$$d_{k\ell} \triangleq 2^{-k+1} e^{-\lambda} (1-p_k) \sum_{j=0}^{\min(k,\ell)} \binom{k}{j} \frac{\lambda^{\ell-j}}{(\ell-j)!} + e^{-\lambda} p_k \frac{\lambda^{\ell+1-k}}{(\ell+1-k)!} U(\ell+1-k) \quad ; \quad k\geq 2,\ \ell\geq 0$$

Then, from the description of the algorithm we easily conclude that the expected lengths $\{L_k\}$ satisfy the following linear system.

$$L_0 = 1 \qquad (4)$$
$$L_k = \sum_{\ell=0}^{\infty} d_{k\ell} L_\ell + 1 \quad ; \quad k \geq 1$$

4.1 System Stability

The stability of the system in (4) is studied via methods as those in [6] and [7]. Given $n \geq 2$ in $\alpha = nA$, the highest Poisson intensity, λ_n^*, for which the system is stable is the throughput of the CRAFH, and $(0, \lambda_n^*)$ is then the stability region of the algorithm. As in [7], λ_n^* is the supremum of all Poisson intensities that provide a nonnegative and bounded solution for the linear system in (4). Directly from the theory developed in [7], we now express the following theorem.

Theorem 1

(i) Given some finite positive integer N, let us consider the following truncated version of the system in (4).

$$x_0 = 1 \qquad (5)$$
$$x_k = \sum_{\ell=0}^{N} d_{k\ell}\, x_\ell + 1 \quad ; \quad 1 \leq k \leq N$$

Given $n \geq 2$ in $\alpha = nA$, let $\lambda_n^*(N)$ be the infinum of the Poisson intensities that do not provide a nonnegative solution for the system in (5). Then, $\lambda_n^*(N)$ is an upper bound to the throughput λ_n^*.

(ii) Given $n \geq 2$ in $\alpha = nA$, given N in (5), given $\lambda \leq \lambda_n^*(N)$, let $\{x_k^* \ ; \ 0 \leq k \leq N\}$ be the nonnegative solution of the system in (5). Then, there exists $\lambda_n^o \leq \lambda_n^*$, such that for every $\lambda \leq \lambda_n^o$, there exist positive constants ε, a, and c, and positive integer $N_o < N$, such that the system in (4) has a solution $\{y_k \ ; \ k \geq 0\}$, which satisfies the following conditions:

$$y_0 = 1 \ , \ 0 \leq y_k \leq (1+\varepsilon) x_k^* \ ; \ 1 \leq k \leq N_o \qquad (6)$$
$$0 \leq y_k \leq ak + c \ ; \ k > N_o$$

The bounds in (6) correspond to a lower bound on the throughput λ_n^*.

Selecting N=20 in (5), and following exactly the same methodology as in [7], we found the following bounds on the throughput λ_n^*, for n=5,10,20.

$$\begin{aligned} 0.576 &\le \lambda_5^* \le 0.577 \\ 0.694 &\le \lambda_{10}^* \le 0.695 \\ 0.787 &\le \lambda_{20}^* \le 0.790 \end{aligned} \tag{7}$$

4.2 Delay Analysis

Let the arriving packets be indexed, according to the order of their arrival time. Let $\mathcal{D}_j$ denote the delay of the jth packet; that is the time from its arrival to its successful transmission. Let $\mathcal{Q}_i$ denote the total number of packets that are successfully transmitted during the first i nonempty sessions. Then, as in [7] and [18], we conclude that $\{\mathcal{Q}_i\}_{i\geq 0}$ is a renewal process, and that the process $\{\mathcal{D}_j\}_{j\geq 1}$ is regenerative with respect to $\{\mathcal{Q}_i\}_{i\geq 0}$, where the common regeneration cycle, $\mathcal{S}$, is the number of successfully transmitted packets during a nonempty session. Let us define,

$$S \triangleq E\{\mathcal{S}\}$$
$$D_S \triangleq E\{\sum_{j=1}^{\mathcal{S}} \mathcal{D}_j\} \tag{8}$$

Then, from the regeneration theorem in [13] and [18], we have that, if $\mathcal{S}$ is nonperiodic and if $S<\infty$ and $D_S<\infty$, then, $\mathcal{D}_j$ converges in distribution to a random variable $\mathcal{D}_\infty$, and, $D \triangleq \lim_{i\to\infty} i^{-1} \sum_{j=1}^{i} \mathcal{D}_j = \lim_{i\to\infty} i^{-1} E\{\sum_{j=1}^{i} \mathcal{D}_j\}$, with probability one

$$D = E\{\mathcal{D}_\infty\} = D_S\, S^{-1} <\infty \tag{9}$$

From the operation of the CRAFH, we conclude that $P_r(\mathcal{S}=1)\neq 0$, so $\mathcal{S}$ is nonperiodic. Given $n\geq 2$ in $\alpha=nA$, and some λ in $(0,\lambda_n^*)$, we compute $S = (1-e^{-\lambda})^{-1}\lambda L<\infty$; where L is as in (2). If we also show that $D_S<\infty$, for each λ in $(0,\lambda_n^*)$, then the regeneration theorem holds, and the parameter D in (9) is then the expected per packet delay induced by the CRAFH. But, if $\mathcal{B}_j$ and $\mathcal{C}_j$ are respectively the access delay and the

contention delay of the jth packet arrival, then $D_j = B_j + C_j$, and,

$$D_S = B + C \tag{10}$$

; where,

$$B \triangleq E\{\sum_{j=1}^{S} B_j\} \quad , \quad C \triangleq E\{\sum_{j=1}^{S} C_j\} \tag{11}$$

Given n and λ in $(0,\lambda_n^*)$, it can be easily found that $B = 2^{-1}S<\infty$, where S is as in (8). Thus, to show then that $D_S<\infty$, it suffices to prove that $C<\infty$, where C represents the expected cumulative per nonempty session contention delay, induced by the CRAFH. Proceeding towards that direction, let us denote by C_k, $k\geq 0$, the expected cumulative per nonempty session contention delay, given that the session starts with k packet transmissions. Then, for $\{L_k\}_{k\geq 0}$ as in the beginning of this section, for p_k as in (1), and for $d_{k\ell}$ as in (3), it is concluded from the operation of the algorithm, that $\{C_k\}_{k\geq 0}$ satisfies the following linear system.

$$\begin{aligned} C_0 &= 0 \\ C_k &= \sum_{\ell=0}^{\infty} d_{k\ell} C_\ell + f_k \quad ; \ k\geq 1 \end{aligned} \tag{12}$$

; where,

$$\begin{aligned} f_1 &= 1 \\ f_k &= (1-p_k) \sum_{i=0}^{k} \sum_{j=0}^{\infty} \binom{k}{i} 2^{-k} e^{-\lambda} \frac{\lambda^j}{j!} (k-i) L_{i+j} + k \quad ; \quad k\geq 2 \end{aligned} \tag{13}$$

Upper and lower bounds of the quantities $\{C_k\}_{k\geq 1}$ in (12) are computed via methods as in theorem 1, and in [7]. Given n, for N_o and $\{x_k^*\}$ as in (6), and for any λ in $(0,\lambda_n^*)$, we find as in [7] that instead of the inequalities in (6), here we have,

$$\begin{aligned} 0 &\leq C_k \leq db(1+\varepsilon)\, x_k^* + dk^2 \quad ; \quad 1\leq k\leq N_o \\ 0 &\leq C_k \leq db(ak+c) + dk^2 \quad ; \quad k>N_o \end{aligned} \tag{14}$$

; where ε, a, c are as in (6), and where d and b are positive and bounded constants. Due to (14), and the equation,

$$C = [1-e^{-\lambda}]^{-1} \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} C_k \tag{15}$$

we conclude that given n, and $\lambda<\lambda_n^*$, C is bounded. Thus, D_S in (10) is then bounded as well, and we can then compute the expected per packet delay, D, as follows.

$$D = D_S\, S^{-1} = [B+C][(1-e^{-\lambda})^{-1} \lambda L]^{-1} = [2^{-1} S+C]\,(1-e^{-\lambda})\,\lambda^{-1} L^{-1} =$$

$$= 0.5 + \lambda^{-1}\left[\sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} L_k\right]^{-1} \sum_{\ell=0}^{\infty} e^{-\lambda} \frac{\lambda^{\ell}}{\ell!} C_{\ell} \tag{16}$$

Upper and lower bounds, D^u and D^{ℓ}, on D are computed as in [7], via the method of truncated linear systems [12]. The specifics of those truncations are routine and are omitted here. The computed bounds D^u and D^{ℓ} are included in table 1, for n=5,10,20 in α =nA, and are identical to the digits shown in the table. In figure 1, D is plotted against λ.

4.3 Steady State Probabilities

Let $\{Z_k^{(i)}\}_{k\geq 0}$, i=0,1,..., be such that if the time interval that corresponds to the kth slot contains i packets, then $Z_k^{(i)}=1$. Otherwise, $Z_k^{(i)}=0$. For every given i, the process $\{Z_k^{(i)}\}_{k\geq 0}$ is regenerative with respect to the renewal sequence formed by the time instants when sessions begin. The regeneration cycle is then the length of a session. Given n and some λ in $(0, \lambda_n^*)$, let L be as in (2), and let $\Delta^{(i)}$ denote the expected number of slots in a session, whose corresponding time intervals contain i packet arrivals. Let $\Delta_k^{(i)}$, $k\geq 0$, denote the expected number of slots, within some session that starts with k packet transmissions, whose corresponding time intervals contain i arrivals. Then, for $d_{k\ell}$ as in (4), and for δ_{ij} being the Kronecker delta, we conclude from the operations of the algorithm that the numbers $\Delta_k^{(i)}$, satisfy the following linear system, for every i, and for λ in $(0,\lambda_n^*)$.

$$\begin{aligned} &\Delta_0^{(i)} = \delta_{i0} \\ &\Delta_k^{(i)} = \sum_{\ell=0}^{\infty} d_{k\ell}\, \Delta_{\ell}^{(i)} + \delta_{ik} \quad ; \quad k\geq 1 \end{aligned} \tag{17}$$

A theorem parallel to theorem 1 can be expressed for the system in (17), where we also have,

$$\Delta^{(i)} = \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} \Delta_k^{(i)} \qquad (18)$$

Thus, we again conclude that $\Delta^{(i)}$ is bounded for every i and for λ in $(0,\lambda_n^*)$, and bounds on $\Delta^{(i)}$ can be computed via methods as in section 4.1. Since L is also bounded, for all λ in $(0,\lambda_n^*)$, the regeneration theorem applies again, to give:

$$\pi_i \triangleq \lim_{k\to\infty} \Pr(Z_k^{(i)}=1) = L^{-1} \Delta^{(i)} \; ; \quad i=0,1,2,.. \qquad (19)$$

Given i, the quantity π_i in (19) is then the steady state probability that a time interval corresponding to a channel slot, contains i packet arrivals. Upper and lower bounds on the π_i are computed routinely via the method of truncated linear systems, as before. We computed those bounds for i=0,1,2,3,4,5, for n=5,10,20 in α=nA, and for various values of λ in the corresponding stability regions of the algorithm. Our results are included in table 2, where the bounds coincide to the corresponding digits in the table. We will use the computed π_i values in the next section, where we will evaluate the performance of the algorithm, when assumption A1 in section 2 is relaxed.

5. Relaxation of Assumption A1 - Multiple Receiver Model

To this point, we assumed lack of interference from transmissions addressing different receivers, which allowed the isolation of each single receiver model. In this section, we will relax this assumption, which departs from reality, as the number, N_R, of receivers in the system increases. Indeed, as N_R increases, the frequency hopping patterns assigned to the receivers at each slot, stop being mutually orthogonal. They become partially correlated instead; thus, transmissions to different receivers interfere with each other. We will study the performance of the CRAFH algorithm in the latter case. We will assume that the codes used by the N_R receivers for their feedback broadcasts are mutually orthogonal; thus, simultaneous broadcasts

from different receivers are received correctly by the users. We note that the relatively low level of the broadcast information allows for such orthogonality, as long as the number, N_R, of receivers does not exceed a relatively large bound.

Let us isolate a single receiver, R_i, in the system, and let us consider the case where in slot T, the receiver locks on to some packet addressed to it. This corresponds to either a single packet transmission to R_i in T, or to multiple transmis ions to R_i in T with capture, and the feedback broadcasted by R_i is then S. In the ab- sence of interferences with simultaneous transmissions addressed to other receivers, ea time that R_i broadcasts S, a single packet has been correctly decoded by it. This is n always the case, however, in the presence of interferences with transmissions to other receivers. Indeed, the Reed-Solomon frequency hopping patterns allow then for correct decoding of a single packet by R_i, only if the number of simultaneous transmissions to other receivers, does not exceed some given number, N. Otherwise, no packet is receive correctly by receiver R_i, while the receiver still broadcasts the feedback S. Thus, whe R_i locks on to some packet, and when more than N packets are simultaneously trans- mitted to receivers different than R_i, then the former packet leaves the system, while it has not been received correctly. The latter event occurs with some positive probability, p, and it clearly results in some traffic loss. The earlier strict definition of the throughput becomes then obsolete, while for the set $\{\pi_i\}$ being as in section 4.3, the probability p is given by the following expression.

$$1-p = \sum_{\sum_{j=0}^{N} j i_j \leq N} (N_R-1)! \prod_{j=0}^{N} \frac{\pi_j^{i_j}}{i_j!} \qquad (20)$$

We now present a definition of throughput for the multiple receiver model, in the presence of interferences from transmissions to different receivers.

<u>Definition</u>

Given n in α=nA, given the number, N_R, of receivers, given p^* such that, $0 < p^* < 1$, the throughput, $\lambda_n^*(N_R,p^*)$, of an algorithm is the maximum Poisson

traffic intensity per receiver, that maintains the value of the probability p (in (20)) below than or equal to p^*.

We note that given some algorithm, the throughput in the above definition depends on the specific choice of the encoding per packet and the length of the frequency hopping pattern (which control the number N in (20)). In section 5.1 below, we will select such patterns, and we will then compute the throughput of the CRAFH algorithm, for various choices of the numbers n, N_R, and p^*.

5.1 Numerical Results

For the encoding of the packets, we will draw from the example on pages 274 and 275 in reference [19]. In particular, we assume that a BCH code of length $2^{10}-1=1023$ bits is used for the encoding of each packet. We assume that the hopping rate is equal to the data bit rate, (i.e. A=1), and we use the Reed-Solomon frequency hopping patterns in section 2. Regarding the length, m, of the frequency hopping patterns and the encoding per packet, we consider the following three cases:

Code 1 : m=1023 and number of information bits 513, which implies N=57 in (20).

Code 2 : m=1023 and number of information bits 748, which implies N=28 in (20).

Code 3 : m=1023 and number of information bits 883, which implies N=14 in (20).

In table 3, we list the values of the throughput, $\lambda_n^*(N_R, p^*)$, induced by the CRAFH algorithm, for the above three codes, and for various choices of the numbers n, N_R, and p^*. We note that similar results can be found, when the hopping rate is slower than the data bit rate (as with the example in [10]).

6. Comments and Conclusions

In spread spectrum systems, the waveform patterns used (frequency hopping patterns in our case) change in time. Consequently, the users and the transmitters must then maintain some degree of synchronization, which is typically accomplished via slotization of the channel time. Thus, the capability for slotted operation is inherent in such systems.

In this paper, we presented and analyzed a limited sensing random access algorithm, for frequency hopping multi-user and multi-receiver spread spectrum systems with slotted transmission channel. We considered the Reed-Solomon frequency hopping patterns described in section 2, and we assumed ternary feedback per slot. The methodology we used for the analysis of the algorithm applies also, when a more general class of Reed-Solomon frequency hopping patterns [11] is used. The throughputs induced by the algorithm, as shown in expression (7) and in table 3, correspond to traffic intensities per receiver. For comparison with random access algorithms used in non spread spectrum systems, and taking into consideration that BCH codes have been used, those throughputs must be divided by the number, q, of the frequency bins in the frequency hopping pattern. We note that for protection against intelligent adversaries (jamming), the bandwidth of the feedback channel must be comparable to the bandwidth of the transmission channel.

We considered a Poisson user model. This model best reflects environments where users are highly mobile. In addition, random access algorithms devised for such a model are robust in the presence of changing traffic. For traffic changes within their stability region, they remain stable, and they induce uniformly good delays. For the frequency hopping spread spectrum systems considered in this paper, and for noisy transmission channels, binary SNS feedback per slot may be considered, instead. Then, per slot, each receiver distinguishes success S (when it locks on to some packet) versus nonsuccess NS (due to the noise in the transmission channel, the receiver can not distinguish between lack of transmission and collision). In the presence of

the above binary feedback and the Poisson user model, stable random access algorithms can be devised, only if each receiver transmits phony packets at certain times. Such a limited sensing random access algorithm can be found in [15].

The algorithm in this paper is a limited sensing random access algorithm. That is, each user is required to monitor the feedback broadcast, only while he is blocked. This property is generally very attractive, and is indispensable in environments where the users are highly mobile.

Concluding, we point out that in the construction of our model, the paper by Davis and Gronemeyer [20] was very helpful. Random access techniques in spread spectrum have also been addressed in [21] and [22].

λ	D for n=5	D for n=10	D for n=20
0.1	1.70501	1.66150	1.64211
0.2	2.07521	1.92846	1.86770
0.3	2.82826	2.40000	2.24196
0.4	4.73012	3.33066	2.91142
0.5	12.8319	5.55964	4.26297
0.57	185.485	---	---
0.6	---	13.7414	7.61929
0.69	---	394.714	---
0.7	---	---	20.8033
0.78	---	---	294.631

Table 1

Expected Delays

$D=D^u=D^\ell$ in the values included.

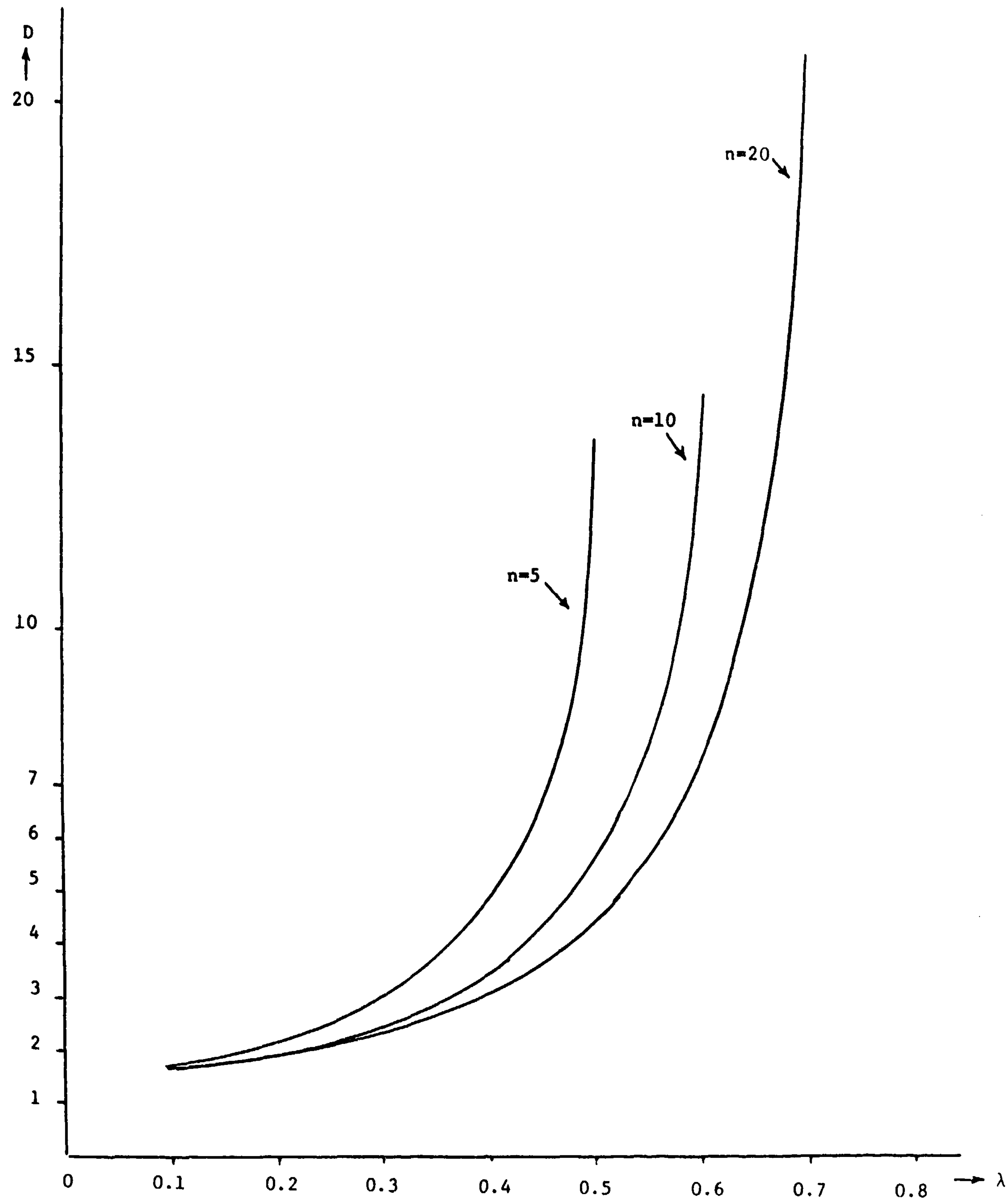

Figure 1

Expected Delays

	λ	π_0	π_1	π_2	π_3	π_4	π_5
n=5	0.1	0.89749614	0.95691416D-01	0.64270581D-02	0.36625470D-03	0.18315279D-04	0.78825077D-06
	0.2	0.78840600	0.18085649	0.26952682D-01	0.33851260D-02	0.36389099D-03	0.33073654D-04
	0.3	0.66922172	0.25174005	0.63250398D-01	0.13148238D-01	0.22704402D-02	0.32536275D-03
	0.4	0.53378950	0.30261246	0.11675354	0.35898610D-01	0.88344977D-02	0.17679347D-02
	0.5	0.37033319	0.32365792	0.18889663	0.81479385D-01	0.26792573D-01	0.70009136D-02
n=10	0.1	0.89882931	0.95136230D-01	0.57479292D-02	0.27451230D-03	0.11560374D-04	0.43857998D-06
	0.2	0.79472288	0.17879821	0.23691979D-01	0.25278709D-02	0.23752667D-03	0.19927777D-04
	0.3	0.68644176	0.24783082	0.54300407D-01	0.96832889D-02	0.15081302D-02	0.20723582D-03
	0.4	0.57201111	0.29808187	0.96950979D-01	0.25699304D-01	0.58575273D-02	0.11604062D-02
	0.5	0.44810270	0.32375644	0.14943933	0.55493960D-01	0.17285700D-01	0.45934195D-02
	0.6	0.30872982	0.[illegible]599813	0.20725737	0.10490765	0.42846940D-01	0.14581594D-01
n=20	0.1	0.89943490	0.94885926D-01	0.54363751D-02	0.23381763D-03	0.86746195D-05	0.29442500D-06
	0.2	0.79748601	0.17791380	0.22245416D-01	0.21554013D-02	0.18380985D-03	0.14431745D-04
	0.3	0.69363870	0.24625118	0.50506797D-01	0.82301477D-02	0.11922063D-02	0.15879362D-03
	0.4	0.58709972	0.29641518	0.89035754D-01	0.21638529D-01	0.46826394D-02	0.92555062D-03
	0.5	0.47657590	0.32399993	0.13480994	0.45861206D-01	0.13801043D-01	0.37501681D-02
	0.6	0.35983412	0.32313168	0.18221743	0.83900370D-01	0.33598815D-01	0.11945134D-01
	0.7	0.23277635	0.28525592	0.22194259	0.13711937	0.71206316D-01	0.32025530D-01

Table 2

Steady State Probabilities, subject to assumption A1.

Code 1	n=5				n=10				n=20			
	$p^*=10^{-3}$		$p^*=10^{-4}$		$p^*=10^{-3}$		$p^*=10^{-4}$		$p^*=10^{-3}$		$p^*=10^{-4}$	
	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$
	337	0.1	304	0.1	346	0.1	313	0.1	350	0.1	317	0.1
	149	0.2	134	0.2	157	0.2	142	0.2	161	0.2	146	0.2
	85	0.3	76	0.3	93	0.3	84	0.3	97	0.3	88	0.3
	52	0.4	47	0.4	60	0.4	54	0.4	64	0.4	58	0.4
	33	0.5	30	0.5	40	0.5	36	0.5	44	0.5	39	0.5
	-	-	-	-	27	0.6	24	0.6	30	0.6	27	0.6
	-	-	-	-	-	-	-	-	21	0.7	18	0.7

Code 2	n=5				n=10				n=20			
	$p^*=10^{-3}$		$p^*=10^{-4}$		$p^*=10^{-3}$		$p^*=10^{-4}$		$p^*=10^{-3}$		$p^*=10^{-4}$	
	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$
	136	0.1	118	0.1	139	0.1	121	0.1	142	0.1	123	0.1
	58	0.2	51	0.2	64	0.2	55	0.2	66	0.2	56	0.2
	34	0.3	29	0.3	38	0.3	32	0.3	39	0.3	34	0.3
	21	0.4	18	0.4	24	0.4	21	0.4	26	0.4	22	0.4
	14	0.5	12	0.5	16	0.5	14	0.5	18	0.5	15	0.5
	-	-	-	-	11	0.6	9	0.6	12	0.6	10	0.6
	-	-	-	-	-	-	-	-	8	0.7	6	0.7

Code 3	n=5				n=10				n=20			
	$p^*=10^{-3}$		$p^*=10^{-4}$		$p^*=10^{-3}$		$p^*=10^{-4}$		$p^*=10^{-3}$		$p^*=10^{-4}$	
	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$	N_R	$\lambda_n^*(N_R,p^*)$
	52	0.1	41	0.1	53	0.1	43	0.1	55	0.1	44	0.1
	23	0.2	18	0.2	24	0.2	19	0.2	25	0.2	19	0.2
	13	0.3	10	0.3	14	0.3	11	0.3	15	0.3	12	0.3
	8	0.4	6	0.4	9	0.4	7	0.4	10	0.4	8	0.4
	5	0.5	4	0.5	6	0.5	5	0.5	7	0.5	5	0.5
	-	-	-	-	4	0.6	3	0.6	5	0.6	4	0.6
	-	-	-	-	-	-	-	-	3	0.7	3	0.7

Table 3

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